BOOK OF ABSTRACTS

GIPSOTECA
DI ARTE ANTICA
MAY 9~11
PISA, ITALY

bsde2024.ec.unipi.it

BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

WORKSHOP **2024**

Participants

Paulin Aubert, Université d'Evry Val d'Essonne, France;
Cyril Benezet, ENSIIE, France;
Giorgia Callegaro, Università di Padova, Italy;
Riccardo Cambini, Università di Pisa, Italy;
Laura Carosi, Università di Pisa, Italy;
Etienne Chevalier, Université d'Evry Val d'Essonne, France;
Fulvia Confortola, Politecnico di Milano, Italy;
Paolo Di Domenicantonio, Università di Pisa, Italy;
Cristina Di Girolami, Università di Bologna, Italy;
Giovanna D'Inverno, Università di Pisa, Italy;
Nicole El Karoui, Université Sorbonne (Paris VI), France
Rachele Foschi, Università di Pisa, Italy;
Alexis Houssard, Université de Paris Cité and BPCE, France;
Erik Iemane, Università di Pisa, Italy;
Monique Jeanblanc, Université d'Evry Val d'Essonne, France;
Celine Labart, LAMA - Université Savoie Mont Blanc, France;
Puya Lafat, KU Leuven, Belgium;
Jerome Lelong, Grenoble INP - UGA, France;
Vathana Ly Vath, ENSIIE, France;
Marie-Amelie Morlais, Université du Mans, France;
Huyen Pham, Université de Paris Cité, France;
Monique Pontier, Institut Mathématique de Toulouse, France;
Dylan Possamaï, ETH Zürich , Switzerland;
Sergio Pulido, ENSIIE, France;
Francesco Russo, ENSTA Paris, Institut Polytechnique de Paris, France;
Simone Scotti, Università di Pisa, Italy;
Xiaolu Tan, the Chinese University of Hong Kong, Hong Kong;
Emanuele Vannucci, Università di Pisa, Italy;

Contents

BSDE 2024
WORKSHOP
BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS
GIPSOTECA DI ARTE ANTICA - MAY 9~11 PISA, ITALY

A sharp càdlàg property for jump diffusions and dynamic programming principle

Alessandro Bondi

Centre de Mathematiques Appliquees - Ecole Polytechnique

Given a standard Brownian motion W and a stationary Poisson point process p with values in R^d, we consider the following SDE of Ito's type:

$$dX_t = b(t;X_t)dt + a(t;X_t)dW_t + \int_{|z|\leq 1} g(X_{t-},t,z)\tilde{N}_p(dt,dz) + \int_{|z|>1} f(X_{t-},t,z)N_p(dt,dz),$$

where $X_s = x \in R^d, 0 \leq s \leq t \leq T$. Here N_p [resp., $\tilde{N}_p$] is the Poisson [resp., compensated Poisson] random measure associated with p. The coefficients b, a and g satisfy Lipschitz-type conditions in the x-variable. We first prove that there exists a sharp version of the pathwise unique strong solution $X = (X_t^{s,x})_{t\geq s}$, i.e., there exists an almost sure event Ω' such that, for every $\omega \in \Omega'$, the map $(s,x,t) \mapsto X_t^{s,x}(\omega)$ is càdlàg in s (for t and x fixed), càdlàg in t (for s and x fixed) and continuous in x (for s and t fixed). This result also solves a problem indicated in Kunita's book on stochastic flows. We then extend our approach to encompass controlled SDEs where the coefficients may depend on a random control function c_t. Using our sharp stochastic flow, we establish rigorously a dynamic programming principle (DPP) for controlled SDEs. The DPP is a fundamental tool in stochastic control with several applications in physics and mathematical finance. Such result has a proof of independent interest and seems to be new even in the case where there is no large-jumps component.
This is a joint work with Enrico Priola

BSDE-based stochastic control for optimal reinsurance in a dynamic contagion model

Claudia Ceci

Universitá di Roma la Sapienza

"We investigate the optimal reinsurance problem in the risk model with jump clustering features introduced in [Brachetta, Callegaro, Ceci and Sgarra, Finance and Stochastic, 2023]. This modeling framework is inspired by the concept initially proposed in [Dassios and Zhao Advances in Applied Probabability, 2011], combining Hawkes and Cox processes with shot noise intensity models. Specifically, these processes describe self-exciting and externally excited jumps in the claim arrival intensity, respectively. The insurer aims to maximize the expected exponential utility of terminal wealth for general reinsurance contracts and reinsurance premiums. We discuss two different methodologies: the classical stochastic control approach based on the Hamilton-Jacobi-Bellman (HJB) equation and a backward stochastic differential equation (BSDE) approach. In a Markovian setting, differently from the classical HJB-approach, the BSDE method enables us to solve the problem without imposing any requirements for regularity on the associated value function. We provide a Verification Theorem in terms of a suitable BSDE driven by a two-dimensional marked point process and we prove an existence result relaying on the theory developed in [Papapantoleon, Possamai, and Saplaouras, Electronic Journal of Probability, 2018] for stochastic Lipschitz generators. After discussing the optimal strategy for general reinsurance contracts and reinsurance premiums, we provide more explicit results in some relevant cases. Finally, we provide comparison results that highlight the heightened risk stemming from the self-exciting component in contrast to the externally-excited counterpart and discuss the monotonicity property of the value function. "
This is a joint work with Alessandra Cretarola.

Shrinkage of filtrations and BSDE

Monique Jeanblanc

LaMME Universitè d'Evry Val d'Essonne

In this paper, we study shrinkage of filtration and we provide an explicit form of the optional projection of a semi martingale on a Brownian filtration, under adequate hypotheses. We apply this result to BSDEs

---

Convergence rate of random walk approximation of Backward SDEs

Céline Labart
LaMME Universitè Savoie Mont Blanc

In this paper, we study in the Markovian case the rate of convergence in Wasserstein distance when the solution to a BSDE is approximated by a solution to a BSDE driven by a scaled random walk as introduced in Briand, Delyon and Mémin (Electron. Commun. Probab. (2001)). This is related to the approximation of solutions to semilinear second order parabolic PDEs by solutions to their associated finite difference schemes and the speed of convergence.
This is a joint work with Philippe Briand, Hannah Geiss and Stefan Geiss.

Optimal switching problems: randomization approach

Marie-Amelie Morlais
Université du Mans

We address a general optimal switching problem over finite horizon for a stochastic system described by a differential equation driven by Brownian motion. The main novelty is the fact that we allow for infinitely many modes (or regimes, i.e. the possible values of the piecewise-constant control process). We allow all the given coefficients in the model to be path-dependent, that is, their value at any time depends on the past trajectory of the controlled system. The main aim is to introduce a suitable (scalar) backward stochastic differential equation (BSDE), with a constraint on the martingale part, that allows to give a probabilistic representation of the value function of the given problem. This is achieved by randomization of control, i.e. by introducing an auxiliary optimization problem which has the same value as the starting optimal switching problem and for which the desired BSDE representation is obtained. In comparison with the existing literature we do not rely on a system of reflected BSDE nor can we use the associated Hamilton-Jacobi-Bellman equation in our non-Markovian framework.
This is a joint work with Marco Fuhrman.

BSDE 2024

WORKSHOP
BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS
GIPSOTECA DI ARTE ANTICA - MAY 9~11 PISA, ITALY

A target approach to Stackelberg games

Dylan Possamaï

ETH Zürich

In this paper, we provide a general approach to reformulating any continuous-time stochastic Stackelberg differential game under closed-loop strategies as a single-level optimisation problem with target constraints. More precisely, we consider a Stackelberg game in which the leader and the follower can both control the drift and the volatility of a stochastic output process, in order to maximise their respective expected utility. The aim is to characterise the Stackelberg equilibrium when the players adopt ?closed-loop strategies?, i.e. their decisions are based solely on the historical information of the output process, excluding especially any direct dependence on the underlying driving noise, often unobservable in real-world applications. We first show that, by considering the-second-order-backward stochastic differential equation associated with the continuation utility of the follower as a controlled state variable for the leader, the latter?s unconventional optimisation problem can be reformulated as a more standard stochastic control problem with stochastic target constraints. Thereafter, adapting the methodology developed by Soner and Touzi or Bouchard, Élie, and Imbert, the optimal strategies, as well as the corresponding value of the Stackelberg equilibrium, can be characterised through the solution of a well-specified system of Hamilton-Jacobi-Bellman equations. For a more comprehensive insight, we illustrate our approach through a simple example, facilitating both theoretical and numerical detailed comparisons with the solutions under different information structures studied in the literature.
This is a joint work with Camilo Hernández, Nicolás Hernández Santibánez, and Emma Hubert.

Weak Dirichlet processes and BSDEs.

Francesco Russo

ENSTA Paris, Institut Polytechnique de Paris

The notion of weak Dirichlet process is the natural extension of semimartingale. Among the examples we find the following.

1. Irregular Markov processes solutions of SDEs with distributional drift with jumps.
2. Solutions of (even continuous) path-dependent SDEs with distributional drift.
3. (Path-dependent) Bessel processes. Identification problem in BSDEs driven by random measure.

The talk puts the emphasis on a BSDE with distributional driver.
This is a joint work with Elena Bandini and Elena Issoglio.

On the regularity of solutions of some linear parabolic path-dependent PDEs

Xiaolu Tan

The Chinese University of Hong Kong

We study a class of linear parabolic path-dependent PDEs (PPDEs) defined on the space of càdlàg paths x, in which the coefficient functions at time t depend on $x(t)$ and $\int_0^t x(s)dA_s$, for some (deterministic) continuous function A with bounded variations. Under uniform ellipticity and Hölder regularity conditions on the coefficients, together with some technical conditions on A, we obtain the existence of a smooth solution to the PPDE by appealing to the notion of Dupire's derivatives. It provides a generalization to the existing literature studying the case where $A_t = t$, and complements our previous work on the regularity of approximate viscosity solutions for parabolic PPDEs. As a by-product, we also obtain existence and uniqueness of weak solutions for a class of path-dependent SDEs.
This is a joint work with Bruno Bouchard.

Author Index

SPONSORSHIP AND PATRONAGE:

Università di Pisa
Dipartimento di Economia e Management

ensiie Paris-Évry

université évry val-d'essonne
université Paris-Saclay

NUS National University of Singapore
Centre for Quantitative Finance
Faculty of Science

CO MAT
Comitato dei Matematici Applicati in Toscana

Fondation du risque
Louis Bachelier

A.M.A.S.E.S.
Association for Mathematic Applied to Social Economic Sciences

LaMME
Laboratoire de Mathématiques et Modélisation d'Évry